Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 30875

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2024.

Fourth Semester

Computer Science and Engineering

MA 8402 — PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A random variable X has the density function $f(x) = K\frac{1}{1+x^2}$, $-\infty < x < \infty$. Find the Value of K.

2. If the probability that a certain kind of measuring device will show excessive drift is 0.05 then what is the probability that the sixth of these measuring devices tested will be the first to show excessive drift?

3. The two dimensional random variable (X,Y) has the joint probability mass function $f(x,y) = \begin{cases} \frac{x+2y}{27}, & x = 0,1,2 \ and \ y = 0,1,2 \\ 0, & \text{otherwise} \end{cases}$

   Are X and Y independent?

4. Can $Y = 5 + 2.8X$ and $X = 3 - 05Y$ be the estimated regression equation of Y on X and X on Y respectively? Explain your answer.

5. A random process $X(t)$ is defined as $X(t) = A\cos(\omega t + \theta)$, where ω and θ are constants and A is a random variable. Determine whether $X(t)$ a wide sense stationary process or not.

6. Define Correlation Ergodic Process.

7. What is the average waiting time of a customer in the 3 server infinite capacity poisson queue if he happens to wait, given that $\lambda = 6/hr$ and $\mu = 4/hr$.

8. What the probability that an arrival to an infinite capacity 3 server poisson queueing system with $\frac{\lambda}{\mu} = 2$ and $P_0 = \frac{1}{9}$ enters the service without waiting?

9. Write the traffic equations of Open Jackson network and Closed Jackson networks.

10. What is the solution if there are c_i servers at node i, among k-nodes?

PART B — (5 × 16 = 80 marks)

11. (a) (I) In a continuous random variable X having the probability density function $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$

Find

(i) $P(0 < X \leq 1)$ (2)

(ii) Cumulative distribution function of X (3)

(iii) Moment generating function of X (3)

(II) If the random variable X takes the value 1, 2, 3 and 4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$. Find

(i) The probability distribution of X (2)

(ii) Mean of X (3)

(iii) Variance of X (3)

Or

(b) (I) Assume that half of the population is vegetarian so that the chance of an individual being a vegetarian is 0.5. Assuming that 200 investigators take sample of 10 individual each to see whether they are vegetarian, how many investigators would you expect to report that

(i) Three people or less were vegetarians (4)

(ii) Exactly four people were vegetarian (4)

(II) If X is normally distributed with mean 11 and standard deviation 1.5, find the number k such that

(i) $P(X > k) = 0.3$ and (4)

(ii) $P(X < k) = 0.09$ (4)

12. (a) (I) The following data represents the number of hours 12 different students watched television during the weekend and the scores of each student who took a test the following Monday.

Hours x	0	1	2	3	3	5	5	5	6	7	7	10
Test score y	96	85	82	74	95	68	76	84	58	65	75	50

(i) Calculate the correlation coefficient and comment (4)

(ii) Calculate the expected test score for a student who watches 9 hours of TV. (4)

(II) Let the joint probability density function for (X,Y) be

$$f(x,y) = \begin{cases} \frac{e^{-y}}{2}, & y > 0, -y < x < y \\ 0, & otherwise \end{cases}$$

Are X and Y dependent? If so, find covariance of (X,Y). (8)

Or

(b) (i) If X and Y are distinct random variables with probability density function $f(x) = e^{-x}, x > 0$ and $f(y) = e^{-y}, y > 0$ respectively. Determine the density function of a random variables $S = \frac{X}{X+Y}$ and $T = X + Y$. are S and T independent? (8)

(ii) A random variable with a mean of 1200 hours and a standard deviation of 250 hours can be used to represent the lifespan of a particular brand of light bulb. Using the Central limit theorem, determine the probability that the average lifespan of 60 light bulbs will be greater than 1250 hours. (8)

13. (a) (i) The number of accidents in a city follows a Poission process with a mean of 2 per day and the number X_i of people involved in the i^{th} accident has the distribution (independent) $P(X_i = k) = \frac{1}{2^k}$, $k \geq 1$. Find the mean and variance of the number of people involved in accidents per week. (10)

(ii) The TPM of a Markov chain with three states 0, 1, 2 is $P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}$ and the initial state distribution of the chain is $P(X_0 = i) = \frac{1}{3}$, $i = 0, 1, 2$. Find $P(X_2 = 2)$. (6)

Or

(b) (I) In a super market there are two brands of rice A and B. A customer buys brand A with probability 0.7 if his last purchase was A and buys brand B with probability 0.4 if his last purchase was B assuming Markov chain model. Obtain

(i) One time TPM P

(ii) 2–step TPM P^2 and

(iii) The stationary distribution.

Hence highlight the proportion of customers who would buy brand A and brand B in the long run. (10)

(II) Determine the mean and variance of the process given that the auto correlation function $R_{XX}(\tau) = 25 + \frac{4}{1 + 6\tau^2}$. Find the mean and variance of the process $\{X(t)\}$. (6)

14. (a) (I) A TV repairman finds that the time spent on his job has an exponential distribution with mean 30 minutes. If the repaired set arrive on an average of 10 per 8–hour day with Poisson distribution.

(i) What is the repairman's idle time each day? (3)

(ii) What is the average queue length? (3)

(iii) Find average number of jobs in the system. (2)

(II) A 2-person barbershop has 5 chair to accommodate waiting customers. Potential customers, who arrive when all 5 chairs are full, leave without entering barbershop. Customers arrive at the average rate of 4 per hour and spend an average of 12 minutes in the barber's chair. Compute P_0, P_1 and L_q. (8)

Or

(b) (I) Arrivals of a telephone in a booth are considered to be poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes.

(i) The telephone department will install a second booth, when convinced that an arrival has to wait on the average for atleast 3 minutes for phone. By how much the flow of arrivals should increase in order to justify a second booth? (4)

(ii) What is the average length of the queue that forms from time to time? (4)

(II) Patients arrive at a clinic having single doctor according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

(i) Find the effective arrival rate at the clinic. (2)

(ii) What is the probability that an arriving patient will not wait? (3)

(iii) What is the expected waiting time until a patient is discharged from the clinic? (3)

15. (a) A car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hours and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. Find the average number of cars waiting in the parking lot, if the time for washing and cleaning a car follows.

(i) Uniform distribution between 8 and 12 minutes (5)

(ii) Normal distribution with mean 12 minutes and standard deviation 3 minutes (5)

(iii) Discrete distribution with values equals to 4, 8 and 15 minutes and corresponding probabilities 0.2, 0.6 and 0.2. (6)

Or

(b) In a book shop, there are 2 sections, one for Engineering books and the other section for Mathematics books. There is only one salesman in each section. Customers from outside arrive at the Engineering book section at a poisson rate of 4 per hour and at the Mathematics book section at a Poisson rate of 3 per hour. The service rates of the Engineering book section and Mathematics book section are 8 and 10 per hour respectively. A customer after service at Engineering book section is equally likely to go to the Mathematics book section or to leave the book shop. However, a customer upon completion of service at Mathematics book section will go the Engineering book section with probability $\frac{1}{3}$ and will leave the book shop otherwise. Find the following.

(i) Joint steady-state probability that there are 3 customers in the Engineering book section and 2 in the Mathematics book section. (6)

(ii) Average number of customers in the book shop. (5)

(iii) Average waiting time of a customer in the book shop. (5)
